UNIT-5

Transient and steady state response :-

→ Systems are with energy stored cannot respond instanteneously and will always exhibit transients whenever they are subjected to i/p or disturbance.

→ The performance characterstics of a control system are specified interms of transient and steady state response to a unit step signal.

→ The transient response specifications are

a. delay time

b. rise time

c. peak time

d. maximum overshoot

e. settling time.

The steady state response specifications are steady state error and settling time.

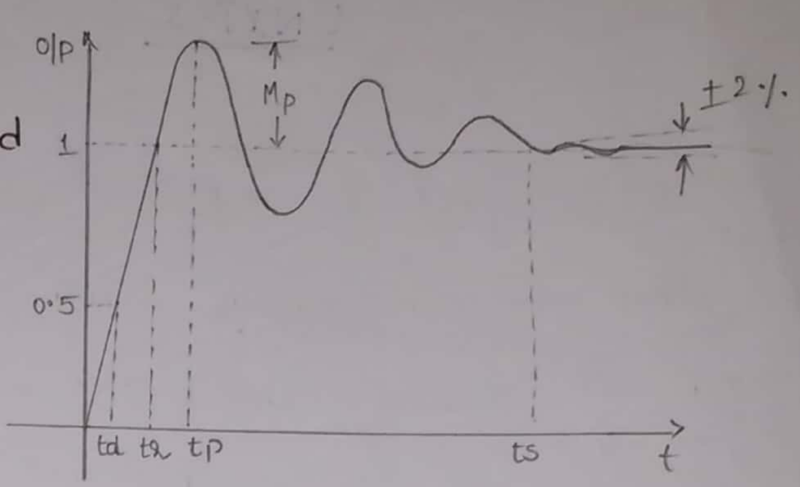
→ An ideal control system must have less delay time, less peak over shoot, less settling time, zero steady state error.

Delay time :-

It is the time required for the response to reach 50% of the final value.

Rise time :-

It is the time required for the response to reach 0 – 100 % (some times 0.1 – 0.9 & 10 % – 90 % of final value)



Peak time :-

It is the time required for the respose to reach the first peak of the overshoot.

Max. Overshoot :-

Max. peak value of the response measured from unity.

$$M_p = c(t_p) - c(\infty)$$

$$\% \, M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100$$

settling time :-

It is the time required for the response to reach and stay within a range of 2 % tolerance of final value.

steady state error :-

The steady state response of stable control system is generally judged by the steady state error due to step and ramp and acceleration equations.

reasons for error may be :—

→ Inability of a system to follow a particular time of o/p

→ Imperfection in system components.

due to frictions.

→ Aging & ~~detoriation~~ deterioration of components.

Whether or not a given system will exhibit steady state err in its response to a given i/p depends on the type of the system.

C.T.C.S

→ The no. of poles of $GH(s)$ at $s=0$

→ Error constants

$$K_p = \lim_{s\to 0} G(s)H(s)$$

$$K_v = \lim_{s\to 0} sG(s)H(s)$$

$$K_a = \lim_{s\to 0} s^2 G(s)H(s)$$

finding ess :—

	type 0	type 1	type 2
step i/p	ess finite $1/1+K_a$	∞	∞
ramp i/p	0	finite = ess = $1/K_v$	∞
parabolic i/p	0	0	ess = finite $1/K_a$

D.T.C.S

→ The no. of poles of $GH(z)$ at $z=1$.

$$K_p = \lim_{z\to 1} GH(z)$$

$$K_v = \lim_{z\to 1} \frac{1-z^{-1}}{T} GH(z)$$

$$K_a = \lim_{z\to 1} \left(\frac{1-z^{-1}}{T}\right)^2 GH(z)$$

	type 0	type 1	type 2
step i/p	ess = finite = $\frac{1}{1+K_a}$	∞	∞
ramp i/p	0	ess = finite = $1/K_v$	∞
parabolic i/p	0	0	ess = finite = $1/K_a$

1 Root locus :-

↳ ~~obto~~ General rules :

1. Obtain the characterstic equation which is in the form of $1 + F(z)$

Let $F(z)$ is written so that the parameter gain k appears as the multiplying factor in the form of

$$1 + \frac{k(z+z_n)(z+z_2) \cdots \cdots (z+z_m)}{(z+P_1)(z+P_2) \cdots \cdots \cdots (z+P_n)} = 0$$

where

$z_1, z_2, z_3 \ldots z_m$ — m - no. of open loop zero's

$P_1, P_2, P_3 \ldots P_n$ — n - no. of open loop poles.

locate open loop poles and zero's

2. ↳ Root locus starts at open loop poles $(k=0)$ and terminates at open loop zero $(k=\infty)$ & at finite open loop zero at "∞".

↳. The no. of root locus branches is equal to p branches. [no. of roots of characterstic equation which is equal to no. of open loop poles].

z = no. of open loop zero's, then the no. of asymptodes are ~~p-m~~ $(p-z)$.

Break Away Breaking Points :

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→ These points may occur on real axis & on complex conjugate case where as the centeroid (intersection of Asymptodes) alway occurs on real axis.

→ Break away point will occurs ~~at real axis~~ between poles, Breaking point will occur ~~at~~ between zero's.

→ If root locus lies between two adjacent open loop poles on the real axis then there exists at least one breakaway point.

→ If root locus lies b/w two adjacent open loop zero's on the real-axis then there will exists at least one breaking point between the zero's.

→ If the root locus lies between one pole and one zero then there may be ~~no~~ breakaway & break in points & there may be both breakpoints exists.

→ The break points can be ~~formed~~ by $1 + F(Z) = 0$

$$1 + K\frac{A(Z)}{B(Z)} = 0$$

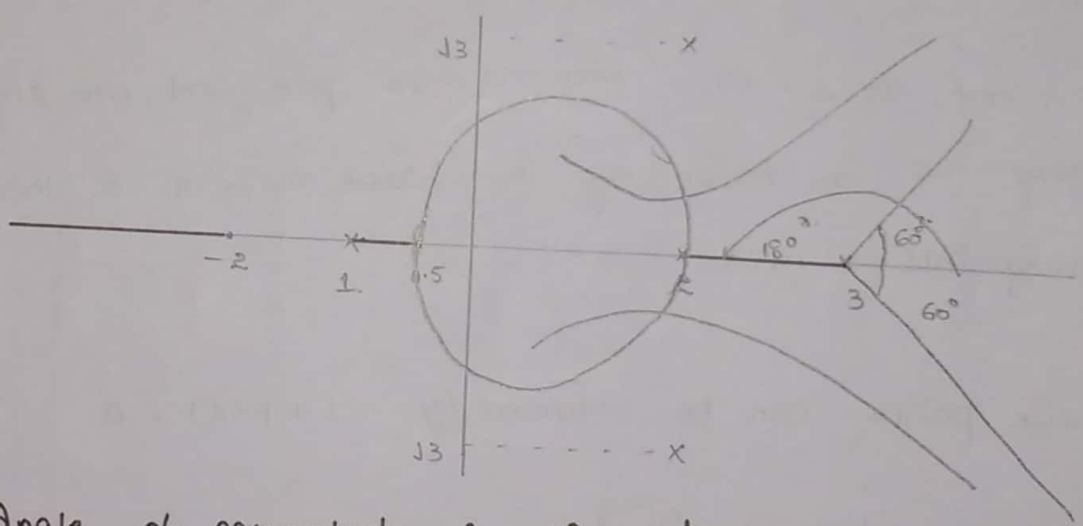
$$K = -\frac{B(Z)}{A(Z)}$$

$$\frac{dK}{dZ} = -\frac{d}{dZ}\left[\frac{B(Z)}{A(Z)}\right] = 0$$

→ Suppose $Z = Z_0$ is a root of $\frac{dK}{dZ}$ now if $K = -\frac{B(Z_0)}{A(Z_0)} > 0$

3. Root locus on real axis :—

- The openloop poles and zero's are laying on real axis determine the portion of the real axis that becomes a part of root locus.
- The conjugate poles and zero's of openloop P.T.F have no effect on the location of root locus on the real axis
- Choose a test point on the real axis and if the total no. of real poles and zero's to the right of the test point is odd then this point lies on the root locus
- Ex: $G(s) = \dfrac{k(z+0.5)(z+2)}{(z+1)(z-3)(z-2)(z-2+j3)(z-2-j3)}$



- Angle of asymptode is given by

$$\frac{\pm 180(2N+1)}{P-Z} \qquad N = 0, 1, 2, 3 \cdots P-Z.$$

intersection of asymptodes

$$\frac{-1+3+2+2-j3+2+j3-(-2.5)}{3} \Downarrow$$

$$\sigma_n = \frac{\Sigma \text{ poles} - \Sigma \text{ zero's}}{P-Z} = 3.5 \quad \left[\because \frac{10.5}{3}\right]$$

5-4

$$\frac{dk}{dz} = \frac{(z-1)(z-0.606)(0.394) - (0.394z)\left((z-0.606)'(z-1)'\right)}{(0.394z)^2}$$

$$\boxed{\frac{uv' - vu'}{v^2}} = \frac{d}{dz}\frac{u}{v}.$$

$\Rightarrow \quad z = \pm 0.778$

$$k = \frac{-(z-1)(z-0.606)}{0.394z} \quad @\ z = 0.778$$

$$= \frac{-(0.778-1)(0.778-0.606)}{(0.394 \times 0.778)} = -0.12456. \quad [-0.12456]$$

$$k = \frac{-(-0.778-1)(-0.778-0.606)}{(0.394 \times -0.778)} \quad @\ z = -0.778$$

1.778×1.384

0.306

$k = \cancel{2.126} \quad \cancel{2.734} \quad 8.027$

if $z = -1$ then $k = 8.152$.

$$\left\{ k = \frac{-(-1-1)(-1-0.606)}{0.394(-1)} = 8.152 \right\}$$

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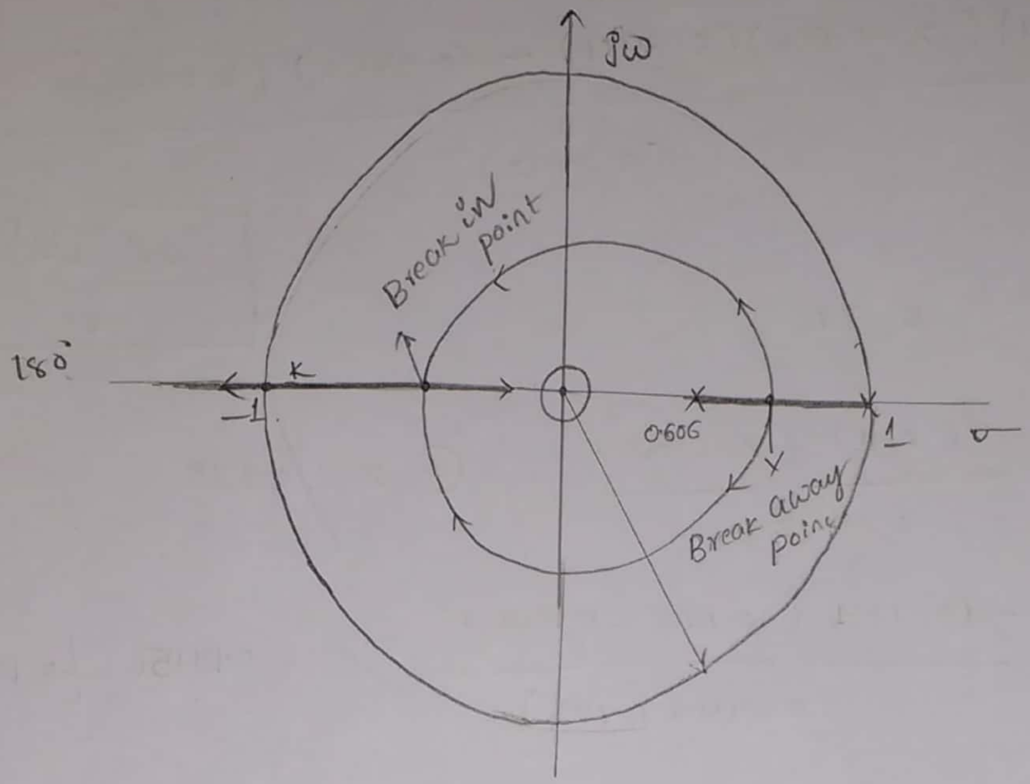

1. No. of Asymptodes = (P−Z) {no. of poles − no. of zero's}

= 2−1

= 1

2. Angle of asymptodes :

$$\frac{180(2N+1)}{P-Z} = 0 \quad \Rightarrow \quad \frac{dK}{dz} = 0$$

$$\frac{K \cdot 0.394z}{(z-1)(z-0.606)} = -1$$

$$\frac{dK}{dz} = \frac{d}{dz}\left[\frac{-(z-1)(z-0.606)}{0.394z \cdot K}\right] = 0$$

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$5 \cdot 5^{4}$

$$G(z) = GD(z) \cdot ZT\left[\frac{1-e^{-TS}}{s} \cdot \frac{1}{s+1}\right]$$

$$= \frac{kz}{z-1}\,(1-z^{-1}) \cdot ZT\left[\frac{1}{s(s+1)}\right]$$

$$Z.T\left[\frac{1}{s(s+1)}\right] \Rightarrow$$

R_1 is the residue at $s=0 \Rightarrow n=1$

$$R_1 = \lim_{s\to 0} \cancel{s} \cdot \frac{1}{\cancel{s}(s+1)} \cdot \frac{z}{z-e^{TS}} = \frac{z}{z-1}$$

TS $s=0$; $T=0.8$

R_2 is the residue at $s=-1$:

$$R_2 = \lim_{s\to -1} \cancel{(s+1)} \frac{1}{s\cancel{(s+1)}} \cdot \frac{z}{z-e^{TS}} = \frac{-z}{z-0.606} \quad [@\ T=0.5\ ;\ s=-1]$$

$$R_1+R_2 = \frac{z}{z-1} - \frac{z}{z-0.606}$$

$$G(z) = k \cdot \frac{\cancel{z}}{\cancel{z-1}} \cdot \frac{\cancel{z-1}}{\cancel{z}} \cdot \left[\frac{z}{z-1} - \frac{z}{z-0.606}\right]$$

$$= \frac{0.394\,kz}{(z-1)\,(z-0.606)}$$

$$\left[Z.T\ \frac{1}{s(s+1)} = \left[\frac{1}{s} - \frac{1}{s+1}\right] \text{ apply Laplace Transform} \right.$$

$$u(t) - e^{-t} \Rightarrow u(KT) - e^{-KT}$$

$$\left. \Rightarrow \frac{z}{z-1} - \frac{z}{z-e^{-T}} = \frac{z}{z-1} - \frac{z}{z-0.606} \right]$$

then "z" is equal to 'z₀' is either a break away & Break in point.

Angle of Departure (θ_d) :-

$$\theta_d = 180 - (\Sigma \text{ all angles lies from all other poles \& zero's to the complex poles}).$$

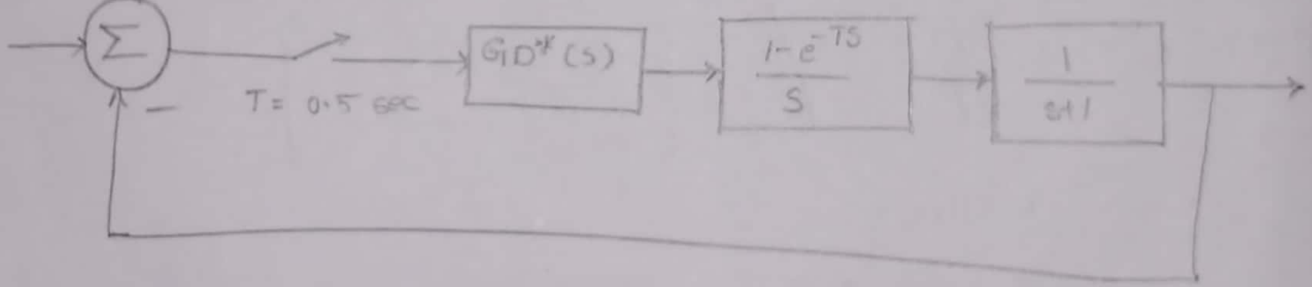
→ The gain 'K' at any specific root location is determined by

$$|F(z)| = 1$$
@ z = required location

problem :-

Determine the critical value of 'K' using root locus for stability

where



$$G_D^*(s) = G_D(z) = \frac{Kz}{z-1}$$ and locate the closed loop poles

corresponding to $K = 2$

sol:-

$$G(z) = G_D(z)\, Z \cdot T \left[\frac{1-e^{-TS}}{S} \cdot \frac{1}{s+1}\right]$$

21/02
5-6 (I)

Effect of sampling period on sampling

A rule of thumb is to sampled is 8 to 10 times is during a cycle of damped sinusoidal oscillations of the output.

→ If it is under damped.

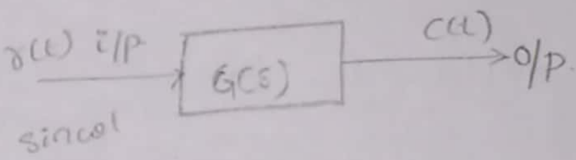
→ For over damped system sample 8–10 times during rise time in step response.

→ For a given value of gain K increasing the sampling period T will make the system less stable.

→ Shorter the sampling period allows the critical value of gain K, for stability to be larger.

→ High sampling frequency gives lower overshoots.

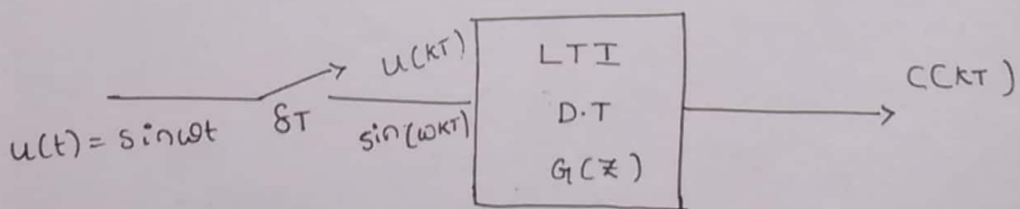
Frequency response :—



Frequency response of G(z) can be obtained by substituting

$z = e^{j\omega T} . G(z)$

if i/p of such a system is linear time invarient



→ if the i/p is sinωt then the steady state output is

$$C(KT)_{ss} = |G(e^{j\omega T})| \sin(\omega KT + \angle G(e^{j\omega T}))$$

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i.e., to obtain the frequency response of $G(z)$, we have to substitute

Bode plots of discrete time signals :-

Since in the z-plane frequency appears as $z = e^{j\omega T}$, if we treat frequency response in the z-plane the simplicity of the logarithm plots will completly lost;

→ This difficulty can be overcome by Bi-linear Transformation by substituting

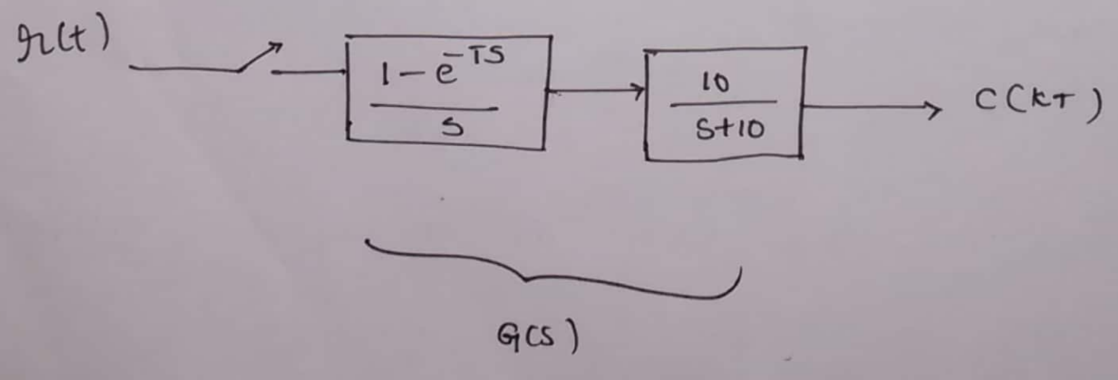
$$z = \frac{1 + \left(\frac{T}{2}\right)\omega}{1 - \left(\frac{T}{2}\right)\omega}$$

→ The region inside of the unit circle in z-plane will be mapped into left hand side of ω-plane.

→ The outside of the unit circle in z-plane will be mapped R.H.S of ω-plane

→ The unit circle will be mapped into $j\omega$ axis.

Ex: Obtain $G(\omega)$ of the following system



21.2.9 5.7

Sol :- $G(z) = Z.T\left[\left(\frac{1-e^{-TS}}{s}\right)\frac{10}{s+10}\right]$

$$= (1-z^{-1})\, Z.T\left[\frac{10}{s(s+10)}\right]$$

R_1 is the residue at $s=0$

$$R_1 = \lim_{s\to 0} \cancel{s}.\frac{10}{\cancel{s}(s+10)}\,\frac{z}{z-e^{TS}}$$

$$= \frac{z}{z-1}$$

$$R_2 = \lim_{s\to -10} \cancel{(s+10)}\,\frac{10}{s\,\cancel{(s+10)}}\cdot\frac{z}{z-e^{TS}}$$

$$= \frac{-z}{z-4.54\times10^{-5}}$$

$$R_1+R_2 = z\left[\frac{1}{z-1} - \frac{1}{z-4.54\times10^{-5}}\right]$$

$$G(z) = \frac{z-1}{\cancel{z}}\cdot\cancel{z}\left[\frac{1}{z-1} - \frac{1}{z-4.54\times10^{-5}}\right]$$

$$= 1 - \frac{z-1}{z-4.54\times10^{-5}}$$

$$= \frac{\cancel{z}-4.54\times10^{-5}-\cancel{z}+1}{z-4.54\times10^{-5}}$$

$$G(\omega) = 1 - \frac{\cancel{1} + \frac{T}{2}\omega - \cancel{1}}{4.54\times10^{-5}}$$

$$1 - \frac{T}{2}\omega - $$

$$= 1 - \frac{\frac{T}{2}\omega}{1 - \frac{T}{2}\omega} \qquad 1 - \frac{\frac{T}{2}\omega}{1 - \frac{T}{2}\omega}$$

$$G(z) = \frac{0.999}{z - 4.54\times10^{-5}}$$

$$G(\omega) = \frac{0.999}{\frac{1+0.5\omega}{1-0.5\omega} - 4.54\times10^{-5}}$$

$$= \frac{0.999}{1+0.5\omega - 4.54\times10^{-5} + 2.27\times10^{-5}\omega}$$

$$G(\omega) = \frac{0.99}{0.99 + 0.5\omega}$$

Lead lag Compensator :-

Lead compensator : It is used for improving stability margin's to increase the bandwidth so the system gives faster response.

→ A system is subjected to high frequency noise because of increased gain

Lag compensation :-

→ It reduces system gain at high frequencies without reducing the gain at low frequencies.

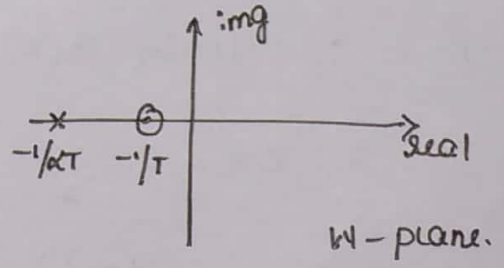
→ Band width is reduced so the system response will be sluggish & will be slow.

→ Steady state accuracy is improved.

→ High frequency noise can be attenuated

→ The T.F of lead compensator is given by

$$G_O(\omega) = \frac{1+T\omega}{1+\alpha T\omega} \qquad 0<\alpha<1$$

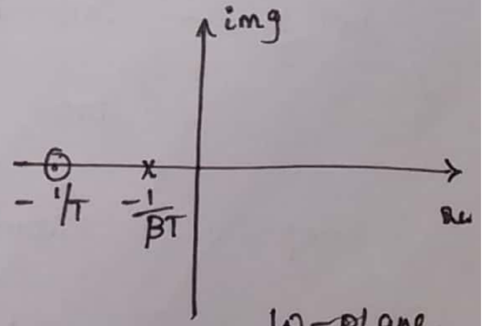


$$W = -\frac{1}{\alpha T} \text{ pole}$$

$$= -\frac{1}{T} \text{ zero}$$

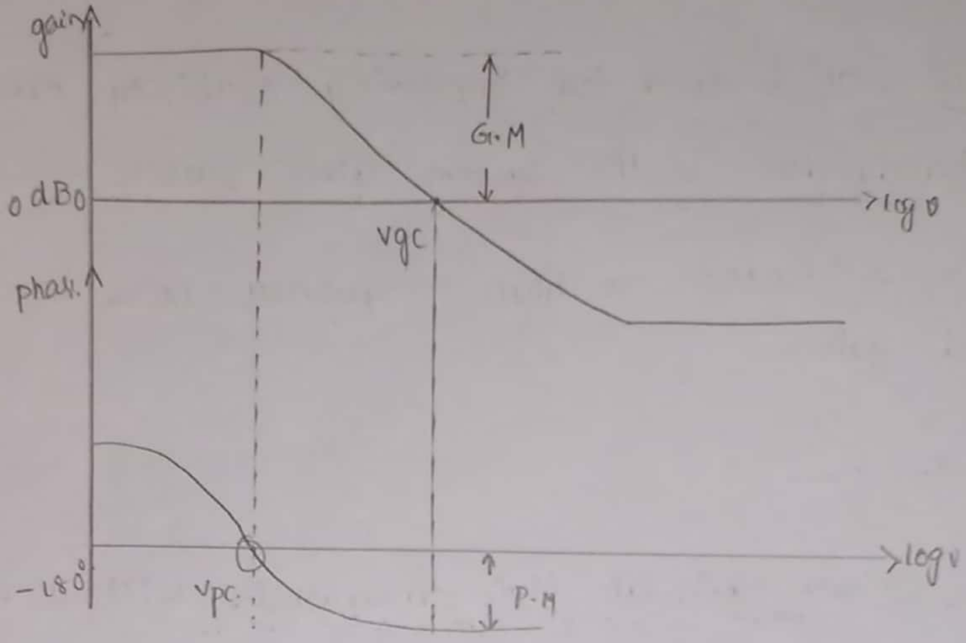
→ Transfer function of lag compensator is

$$G_D(\omega) = \frac{1+T\omega}{1+\beta T\omega} \qquad \beta > 1$$



pole – $\omega = -1/\beta T$

$\omega = -1/T$



→ For example we have to design a compensator in order to improve gain & phase margin

→ Let us take the phase margin of the uncompensated system is 30° and then specified phase margin 50°

Design procedure in ω-plane :-

step-1 : obtain G(z) then obtain G(ω) by substituting

$$z = \frac{1 + \frac{T\omega}{2}}{1 - \frac{T\omega}{2}}$$

step 2 :- Substitute $\omega = jW_\omega$ (W_ω frequency) into G(ω) and plot bode a diagram of $G(jW_\omega)$

step 3 :- From the bode diagram read the phase margin and gain margin

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Step-4 : Assuming the low frequency gain of the discrete time controller $G_D(\omega)$ as unit, determine the system gain by satisfying the requirement of a static error constant and determine the poles and zero's.

Step 5 : Transform the controller T.F $G_D(\omega)$ into $G_D(z)$ by substituting $\omega = \frac{2}{T} \frac{z-1}{z+1}$

Designing Lead compensator in ω-plane :-

→ Assume the transfer function of lead compensator as

$$\frac{1+T\omega}{1+\alpha T\omega} \cdot K_D \qquad 0<\alpha<1$$

→ Find the plant transfer function $G(\omega)$

→ Then $G_D(\omega)\, G(\omega)$ will become an open loop transfer function

$$G_D(\omega)\, G(\omega) \longrightarrow \frac{\text{O.L.T.F}}{K_D \left(\frac{1+T\omega}{1+\alpha T\omega}\right) G\omega}$$

$$= \frac{1+T\omega}{1+\alpha T\omega}\, G'(\omega)$$

Finding the value of "K_D" :-

→ The K_D value is determined using the static velocity error constant

→ Now draw the bode plot of $G'(\omega)$ then evaluate the margin

→ Evaluate the necessary face lead 'ϕ_m' to be added to the system. ϕ_m = desired P.M − P.M of $G'(\omega)$.

→ add $5^\circ - 12^\circ$ to ϕ_m let the new value is 'πm'

Finding 'α":

→ $\sin \phi m = \frac{1-\alpha}{1+\alpha}$

→ Finding the new gain cross over frequency (V_m)

→ In the magnitude plot of $G'(\omega)$ draw a horizon line of height $-20 \log \frac{1}{\sqrt{\alpha}}$

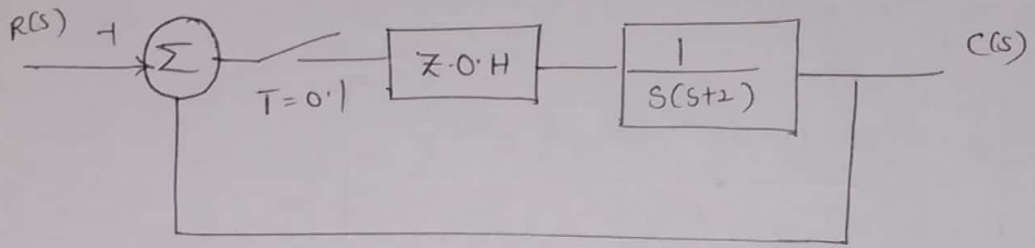
→ Then mark the intersection of this line with magnitude

→ The frequency corresponding to the intersection point gives the new gain cross over frequency (V_m).

Finding "T" :-

→ The T can be found by $V_m = \frac{1}{T\sqrt{\alpha}}$

Design a controller for the system which shown in below. Use the 8/3/10 bode diagram approach in the w-plane. The design specifications are that the phase margin be $55°$, the gain margin be atleast 10 dB and the static velocity error constant be $5 \sec^{-1}$ The sampling period is specified as 0.1 sec & $T = 0.1$. After the controller is designed



Sol :-

$$G(z) = z \cdot T\left[\frac{1-e^{TS}}{s} \cdot \frac{1}{s(s+2)}\right]$$

$$= (1-z^{-1})\, z \cdot T\left[\frac{1}{s^2(s+2)}\right]$$

R_1 is the residue at $s=0$; $n=2$

$$R_1 = \frac{1}{(2-1)!} \lim_{s \to 0} \frac{d}{ds} \cancel{s^2} \cdot \frac{1}{\cancel{s^2}(s+2)} \frac{z}{z-e^{TS}}$$

$$= \lim_{s \to 0} \frac{-z\left[(s+2)(-Te^{TS}) + (z-e^{TS}) \cdot 1\right]}{(s+2)^2 (z-e^{TS})}$$

$$= \frac{-z\left(2(0.1) + (z-1)\right)}{4(z-1)^2} = \frac{-z\left[-0.2+z-1\right]}{4(z-1)^2}$$

$$= \frac{z\left[1.2-z\right]}{4(z-1)^2}$$

R_2 is the residue at $s = -2$

$$R_2 = \lim_{s \to -2} (s+2) \frac{1}{s^2 (s+2)} \cdot \frac{z}{z - e^{Ts}}$$

$$\Rightarrow \frac{z}{4(z - e^{-0.2})} = \frac{z}{4(z - 0.817)}$$

$$G(z) = (1 - z^{-1}) \left[\frac{z(1.2 - z)}{4(z-1)^2} + \frac{z}{4(z - 0.8187)} \right]$$

$$= \frac{\cancel{z-1}}{z} \cdot \frac{\cancel{z}}{4} \left[\frac{(1.2 - z)(z - 0.8187) + (z-1)^2}{(z-1)^{\cancel{2}} (z - 0.8187)} \right]$$

$$= \frac{1}{4} \left[\frac{1.2z - z^2 - 0.9824 + 0.8187 + z^2 - 2z + 1}{(z-1)(z - 0.8187)} \right]$$

$$= 0.00468 \left[\frac{z + 0.94}{(z-1)(z - 0.818)} \right]$$

$$G(\omega) = 0.00468 \left[\frac{\frac{1 + 0.05\omega}{1 - 0.05\omega} + 0.935}{\left(\frac{1 + 0.05\omega}{1 - 0.05\omega} \right) \left(\frac{1 + 0.05\omega}{1 - 0.05\omega} - 0.8187 \right)} \right]$$

$$\Rightarrow 0.00468 \frac{(1 + 0.05\omega + 0.935 - 0.0467 / 1 - \cancel{0.05}\,\omega)}{\left(\frac{1 + 0.05\omega - 1 + 0.05\omega}{1 - \cancel{0.05}\,\omega} \right) \left(\frac{1 + 0.05\omega - 0.8187 + 0.04\omega}{1 - 0.05\omega} \right)}$$

$$= \frac{0.00468\,(1.935 + 0.0033w)\,(1-0.05w)}{0.1w\,(0.1813 + 0.091w)}$$

$$= \frac{0.00468 \times 1.935\,(1 + 0.0017w)(1-0.05w)}{0.1 \times 0.1813 \times w\,(1 + 0.5019w)}$$

$$= \frac{0.5\,(1+0.0017w)\,(1-0.05w)}{w\,(1+0.5019w)}.$$

$$G_D(w) = K_D\,\frac{1+\tau w}{1+\alpha\tau w} = K_D\,\frac{1+\frac{w}{a}}{1+\frac{w}{b}}$$

$\tau = 1/a$ & $\alpha\tau = 1/b$

open loop transfer function is

$$G_D(w)\,G(w) = K_D \cdot \frac{1+(w/a)}{1+(w/b)}\;\frac{0.5\,(1+0.0017w)\,(1-0.05w)}{w\,(1+0.5019w)}$$

The required state velocity constant K_v is $5\,\text{sec}^{-1}$

$$K_v = \lim_{w \to 0} w\,G_D(w)\,G(w)$$

$$5 = \lim_{w\to 0} \cancel{w}.K_D\,\frac{1+(w/a)}{1+(w/b)} \cdot \frac{0.5\,(1+0.017w)\,(1-0.05w)}{\cancel{w}\,(1+0.0501\,9w)}$$

$\Rightarrow 0.5\,K_D = 5$

$K_D = 10$

$$G'(\omega) = \frac{(10)\,0.5\,(1+0.0017\omega)(1-0.05\omega)}{\omega(1+0.5019)\omega}$$

Initial ~~gain~~ magnitude of gain plot $= 20 \log 5 = 13.97$ dB

	Term	Cornerfrequency (in rad/sec)	Slope of the term	Slope of the plot.
1	gain 5	—	—	—
2	ω	0	-20 dB/dec	-20 dB/dec
3	$1+0.5016\omega$	$1.993 \simeq 2$	-20 dB/dec	-40 dB/dec
4.	$1-0.05\omega$	20	$+20$ dB/dec	-20 dB/dec
5	$1+0.00166\omega$	600.2	$+20$ dB/dec	0 dB/dec

Corner frequency:

$$= \frac{1}{0.00166} ; \frac{1}{0.05} ; \frac{1}{0.5016}$$

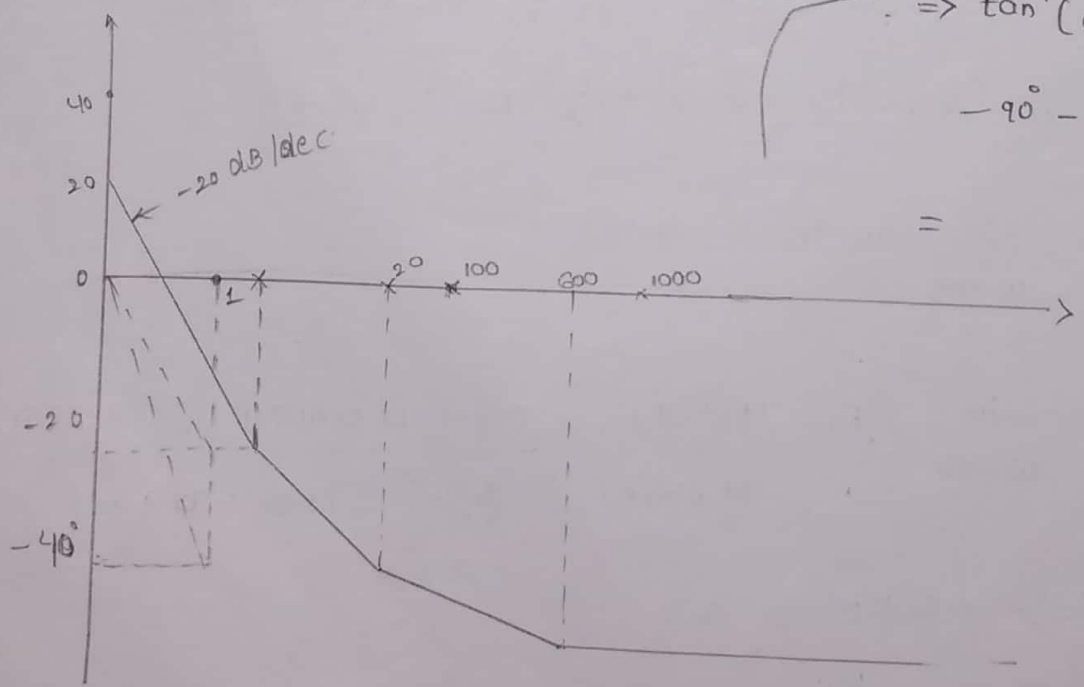
$$= 600.24 , 20 , 1.993$$

$$G'(j\upsilon) = \frac{5(1+j0.001666\upsilon)(1-j0.05\upsilon)}{j\upsilon(1+j0.5016\upsilon)}$$

$\Rightarrow \tan^{-1}(0.001666\upsilon) + \tan^{-1}(-0.05\upsilon)$

$-90^\circ - \tan^{-1}(0.5016\upsilon)$

$=$



Determine the range of k for stability of the system described by $c(k-3) - 2c(k-2) + 1.5kc(k-1) + k\,c(k) = x(k-1) - 2x(k)$

Sol :-

$$c(k-3) - 2c(k-2) + 1.5kc(k-1) + k\,c(k) = x(k-1) - 2x(k)$$

$$\Rightarrow z^{-3}C(z) - 2z^{-2}C(z) + z^{-1}1.5kC(z) + kC(z) = z^{-1}X(z) - 2X(z)$$

$$\Rightarrow C(z)\left[z^{3} - 2z^{-2} + 1.5k + k\right] = X(z) - \left[z^{-1} - 2\right]$$

$$\Rightarrow \frac{C(z)}{X(z)} = \frac{z^{-1} - 2}{z^{-3} - 2z^{-2} + 1.5kz^{-1} + k}$$

$$= \frac{z^{2} - 2z^{3}}{z^{3}k + 1.5kz^{2} - 2z + 1}$$

0	z	z^2	z^3
1	-2	$1.5k$	k
k	$1.5k$	-2	1
$1.5+2k$	$-2-1.5k^2$	$1-k^2$	

① $1 < k$.

$1.5+2k > 1-k^2$

⑪ $P(1) = k + 1.5k - 2 + 1 > 0$

$2.5k - 1 > 0$.

$k > \frac{1}{2.5} = 0.4$

$P(-1) = 2.5k - 1 < 0$

$k < \frac{1}{2.5} = 0.4$

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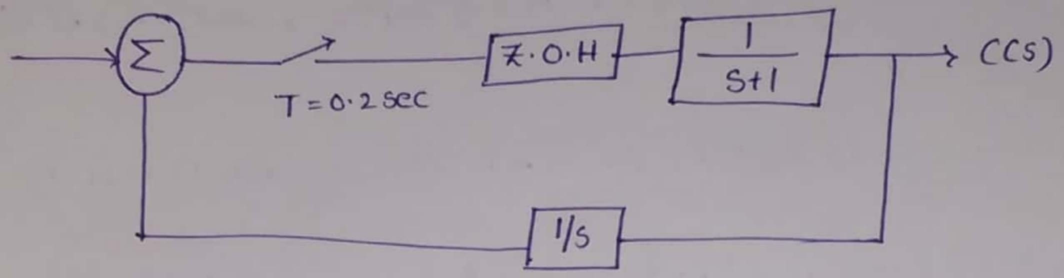
Using bilinear transformation test the stability of following



Sol :-

$$P.T.F = \frac{G(z)}{1+GH(z)}$$

$$GH(z) = z.T\left[z.O.H. \frac{1}{s+1} \cdot \frac{1}{s}\right]$$

$$= (1-z^{-1})\ z.T\left[\frac{1}{s^2(s+1)}\right]$$

R_1 is the residue at $s=0$ & $n=2$

$$\Rightarrow \lim_{s\to 0} \frac{d}{ds} \cancel{s^2} \cdot \frac{1}{\cancel{s^2}(s+1)} \cdot \frac{z}{z-e^{Ts}}$$

$$\Rightarrow \lim_{s\to 0} \frac{-z\left[(s+1)(-Te^{Ts}) + (z-e^{Ts}) \cdot 1\right]}{(s+1)^2 (z-e^{Ts})^2}$$

$$\Rightarrow \frac{-z\left[-0.5 + z-1\right]}{(z-1)^2}$$

$$\Rightarrow \frac{-z\left[z-1.5\right]}{(z-1)^2} = \frac{z.\left[1.5-z\right]}{(z-1)^2}$$

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R_2 is the residue at $s=-1$.

$$\lim_{s \to -1} \cancel{(s+1)} \cdot \frac{1}{\cancel{(s+1)} \cdot s^2} \cdot \frac{z}{z-e^{Ts}}$$

$$\Rightarrow \frac{z}{z-e^{-0.5}} \Rightarrow \frac{z}{z-0.606}.$$

$$GH(z) = \frac{(z-1)}{\cancel{z}} \cdot \cancel{z} \left[\frac{1.5-z}{(z-1)^2} + \frac{1}{z-0.606} \right]$$

$$\Rightarrow 1+GH(z) = 0$$

$$= 1 + \frac{1.5-z}{z-1} + \frac{z-1}{z-0.606} = 0$$

$$\Rightarrow (z-1)(z-0.606) + (1.5\cancel{z}-z)(z-0.606) + (z-1)^2 = 0$$

$$\Rightarrow z^2 - z - 0.606z + 0.606 + 1.5z - z^2 - 0.909 - z^2 + z^2 - 2z + 1 = 0$$

$$\Rightarrow z^2 - 1.5z + 0.697 = 0$$

substitute $z = \frac{\omega+1}{\omega-1}$

$$\Rightarrow \left(\frac{\omega+1}{\omega-1}\right)^2 - 1.5\left(\frac{\omega+1}{\omega-1}\right) + 0.697 = 0$$

$$\Rightarrow \omega^2 + 2\omega + 1 - 1.5\omega^2 + 1.5 + 0.697\omega^2 + 0.697 - 1.394\omega = 0$$

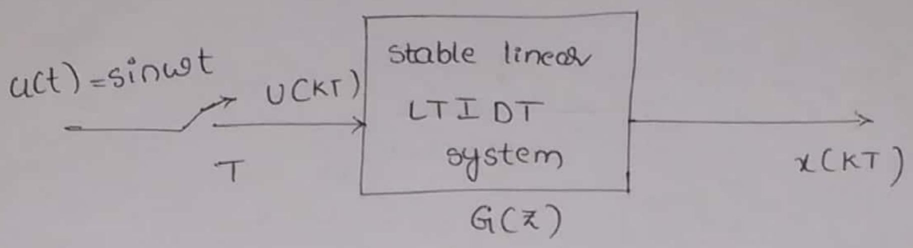
$$\Rightarrow 0.197\omega^2 + 0.606\omega + 3.197 = 0$$

w^2	0.197	3.197
w^1	$0.606w$	0
w^0	3.197	

There is no sign changes are in the given above system.

So the system is stable system.

Response of a linear-Time Invariant Discrete-Time System to a sinusoidal input (or) frequency Response :- (5.14)



→ Frequency Response of G(z) can be obtained by substituting $z = e^{j\omega T}$ into G(z)

if i/p $u(t) = \sin\omega t$

$u(KT) = \sin\omega KT$

Then steady-state output $x(KT)_{ss} = \left|G(e^{j\omega})\right| \sin(\omega KT + \angle G(e^{j\omega}))$

→ i.e, to obtain the frequency response of G(z), we need only to substitute $e^{j\omega T}$ for z in G(z). The function $G(e^{j\omega})$ is commonly called the "Sinusoidal pulse transfer function".

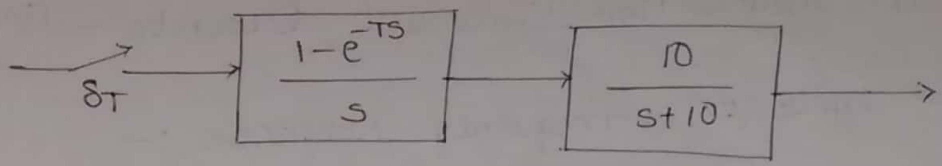
Bilinear - Transformation & ω-plane :-

→ Since in the z-plane frequency appears as $z = e^{j\omega T}$ if we treat frequency response in the z-plane, the simplicity of the Logarithmic plots will be completly lost.

→ The difficulty can be overcome by Bi-linear transformation which transfer z-plane into ω-plane by

$$z = \frac{1 + (T/2)\omega}{1 - (T/2)\omega}$$

Ex :-



Obtain $G(\omega)$

SOL :- $G(\omega) = 9.241 \cdot \dfrac{1-0.05\omega}{\omega+9.241}$

Lag Compensator designing :-

i) Assume lag compensator $G_D(\omega) = K_D \dfrac{1+\tau\omega}{1+\beta\tau\omega}$ $\beta > 1$

ii) open-loop transfer function will become

$$G_D(\omega)\, G(\omega) = K_D \frac{1+\tau\omega}{1+\beta\tau\omega} \cdot G(\omega) = \frac{1+\tau\omega}{1+\beta\tau\omega} \; G_1(\omega)$$

iii) Draw the bodeplot of $G_1(\omega)$ by $\omega = j\nu$ substitution.

iv) K_D is determined by designed static velocity error constant condition

v) Find new gain crossover frequency : The frequency point.

Where phase angle of $G_1(\omega) = -180 +$ required phase margin

required phase margin = specified phase margin + $(5-12^\circ)$

vi) at this new gain cross over frequency the magnitude will be equal to $-20 \log \beta_0$. from this determine β.

vii) finding τ

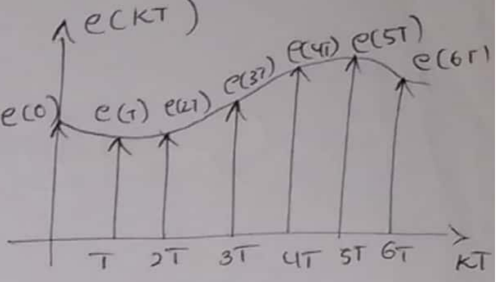
Locate corner-frequency, one decade below the new gain crossover frequency

V = one decade lower to new gain cross over frequency then $V = 1/\tau$

Ex: PTF of Digital PID Controller : ($G_D^*(s)$ & $G_D(z)$) (5.15)

→ In many industries (plants) the digital computer controls several loops may be handled by PID control schemes.

→ Analog PID controller



$$m(t) = K\left[e(t) + \frac{1}{T_i}\int_0^t e(t)\,dt + T_d\frac{de(t)}{dt}\right] \quad \text{——①}$$

→ approximate discrete eqn ①

integration $\xrightarrow{\text{approximated}}$ Trapizoidal summation

derivation $\xrightarrow{\text{approximated}}$ Two point difference

$$① \Rightarrow m(KT) = K\left[e(KT) + \frac{T}{T_i}\left\{\frac{e(0)+e(T)}{2} + \frac{e(T)+e(2T)}{2} + \cdots \frac{e(K-1)T+e(KT)}{2}\right\} + T_d\left[\frac{e(KT - e((K-1)T)}{T}\right]\right]$$

$$m(KT) = K\left[e(KT) + \frac{T}{T_i}\sum_{h=1}^{K}\frac{e((h-1)T)+e(hT)}{2} + \frac{T_d}{T}\left[e(KT) - e(K-1)T\right]\right]$$

$$M(z) = K\left[E(z) + \frac{T}{T_i}\frac{1}{1-z^{-1}}\, z\left[\frac{e((h-1)T)+e(hT)}{2}\right] + \frac{T_d}{T}\left(e(KT) - e((h-1)T)\right)\right]$$

$$M(z) = K\left[E(z) + \frac{T}{T_i}\frac{1}{1-z^{-1}}\, z\left\{\frac{e((h-1)T)+e(hT)}{2}\right\} + \frac{T_d}{T}\left(E(z) - z^{-1}E(z)\right)\right)$$

$$= K\left[E(z) + \frac{T}{T_i}\frac{(1+z^{-1})}{2}\frac{E(z)}{1-z^{-1}} + \frac{T_d}{T}(1-z^{-1})E(z)\right]$$

$$= K\left[1 - \frac{T}{2T_i} + \frac{T}{T_i}\frac{1}{1-z^{-1}} + \frac{T_d}{T}(1-z^{-1})\right]E(z)$$

$$\boxed{M(z) = K_P + \frac{K_I}{1-z^{-1}} + K_D(1-z^{-1})E(z)}$$

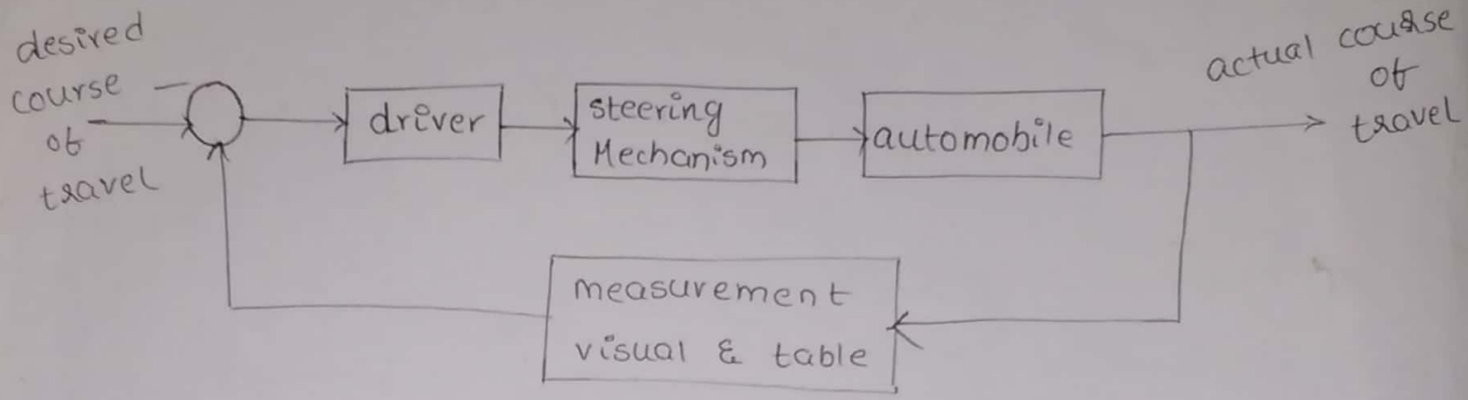
where $K_P = K - \frac{KT}{2T_i}$; $K_I = \frac{KT}{T_i}$ $K_D = \frac{KT_d}{T}$

$$\boxed{G_D(z) = K_P + \frac{K_I}{1-z^{-1}} + K_D(1-z^{-1})} \leftarrow \text{PTF of Digital PID (positional form)}$$

Scanned by CamScanner

Block diagram of digital control system where it employed:-

1. Automobile steering control system :



2. Aircraft flight path control system using GPS :-