DIGITAL CONTROL SYSTEMS

① ①

Why Digital Control :-

→ The Control systems so far we studied deals with the signal at every point in the system is a continuous function of time. In particular the controller elements are such that the controller produces continuous time control signals from continuous time input signals.

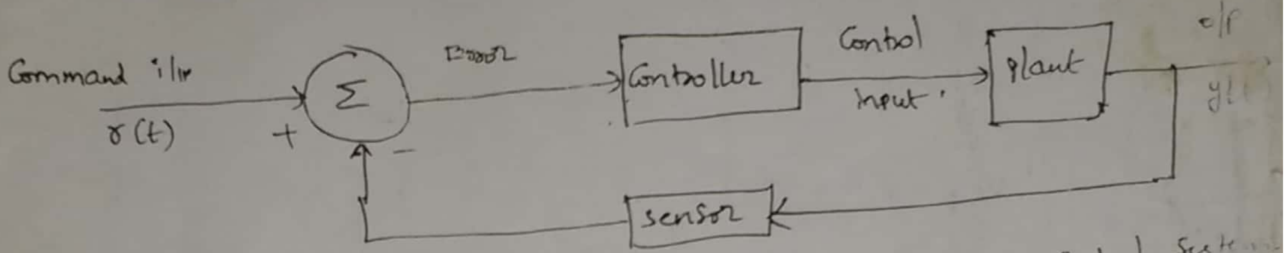

fig: A typical closed loop continuous time Control System

→ The construction of control scheme of complex control function using Analog elements becomes more complex. It leads to loss of flexibility, adaptability and optimality. These drawbacks overcome in Digital control system, because these uses Digital computers.

Advantages of Digital control system:-

→ The use of Digital Control
 1. increases reliability
 2. offers flexibility, adaptability, and optimality

→ Micro controllers can be equipped with a sufficiently large memories to handle a large amount of data in a complex control process.

→ The speed of the micro controllers is also very high (upto 2 MHz)

→ This micro controller reduces the number of components and the complexity of controller.

→ A Digital controller also has the versatility that its control function can be easily modified by changing a few program instructions.

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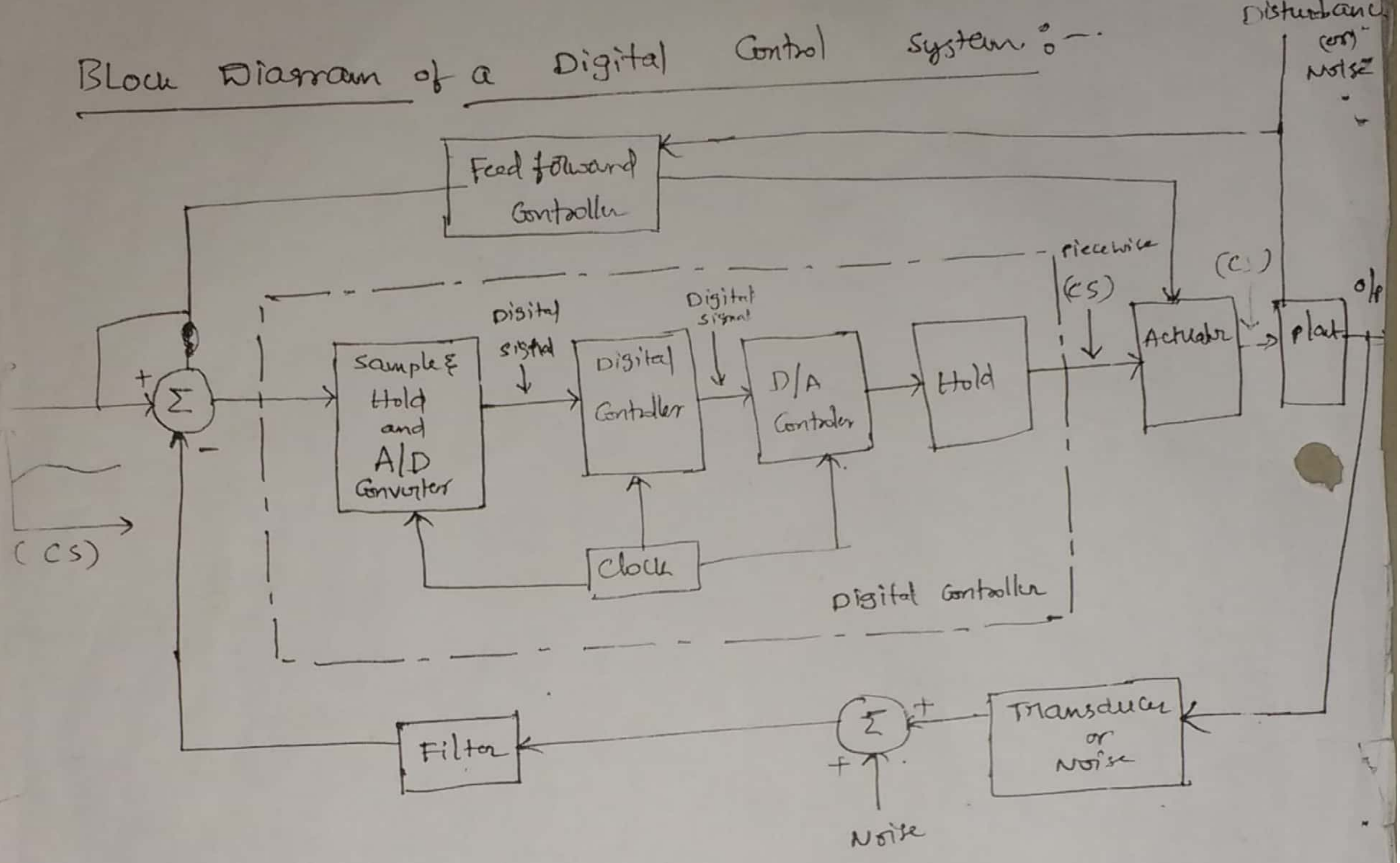
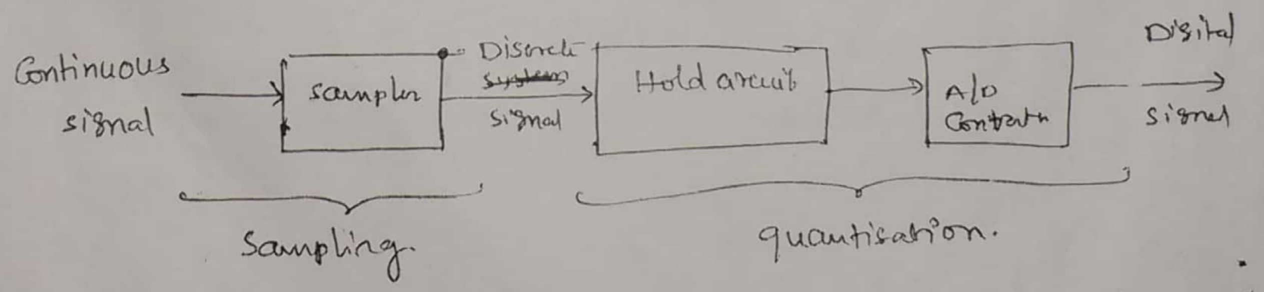
→ These Computers are able to receive and manipulate several inputs, so a digital control system can often be a multivariable system.

→ Block Diagram of a Digital Control system :-



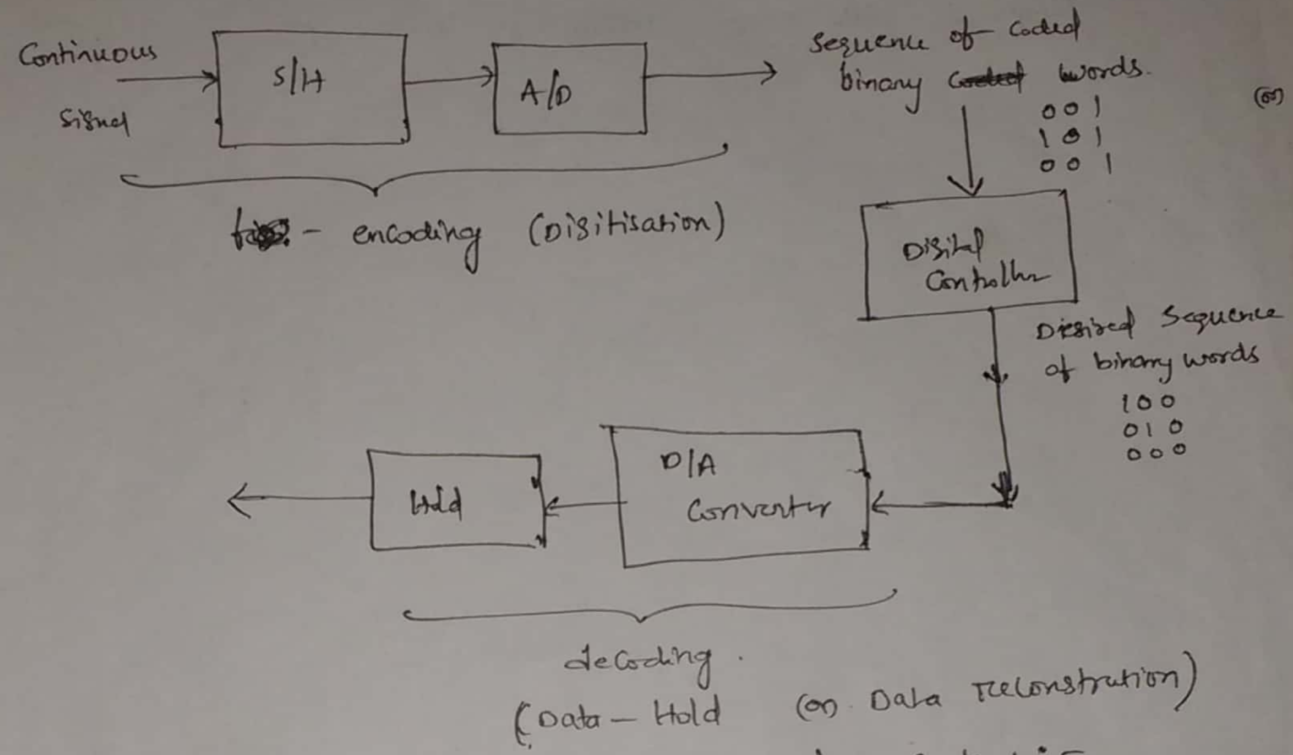
→ output of plant is a Continuous signal

→ The Error signal is Converted into digital form by the sample and Hold circuit and the A/D converter.



→ The Digital signal is now processed through a set of instructions or Algorithm in Digital controller and produces a new Digital signal.

→ The ~~An~~ D/A Converter and Hold circuit produces a piecewise continuous signal. (Re Construction of signal) 2



⇒ <u>Components of a typical Digital Control Systems</u> :-

1. <u>Sample - and - Hold</u> (S/H ckt) (discretisation)
2. <u>Analog to Digital Converter</u> (A/D converter) (quantisation (or) digitisation (or) encoding).
3. <u>D/A Converter</u> — (decoding)
4. <u>Plant or Process</u>
5. <u>Transducer</u>

1. <u>Sample and Hold circuit</u> :-

→ S/H circuit receives Analog signal and hold this signal at a constant value for a specified period of times

→ A sampler converts analog signal into a train of amplitude - modulated pulses.

→ The Hold circuit holds the value of the sampled pulse signal over a specified period of time.

→ without S/H circuit A/D Conversion is impossible.

→ In practise, the sampling duration is very short compared with the sampling period T.

1.3.

3

Introduction to Signals :-

→ signals are detectable quantities used to convey information about time-varying physical phenomena.

Ex human speech., stock prices. &
temperature, pressure, voltage, current

→ mathematically signals are modeled as functions of one or more independent variables (time, frequency or spatial coordinates).

⇒ classification of signals :-

1. Continuous - time and Discrete - time signals. (Based on time)
2. analog and Digital signals. (Based on Amplitude)
3. Periodic and Non periodic signals. (Based on repetitive nature).
4. energy and power signals.
5. Deterministic and probabilistic signals.
6. even and odd signals.
7. ~~causal and non-causal signals.~~

⇒ Continuous time and Discrete time signals :-

→ if a signal is defined for all values of the independent variable t, it is called a Continuous-time (CT) signal.

→ These signals vary continuously with time and have known magnitudes for all the time instants.

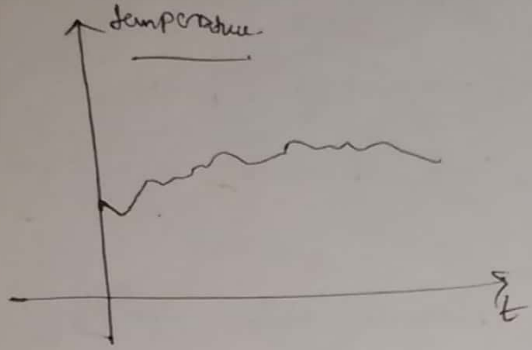
→ If a signal is defined only at discrete values of time (integer ~~values~~ multiples of time) it is called a discrete time signal.

→ If a continuous signal is defined with $x(t)$ then a Discrete time signal is represented with $x(kT)$

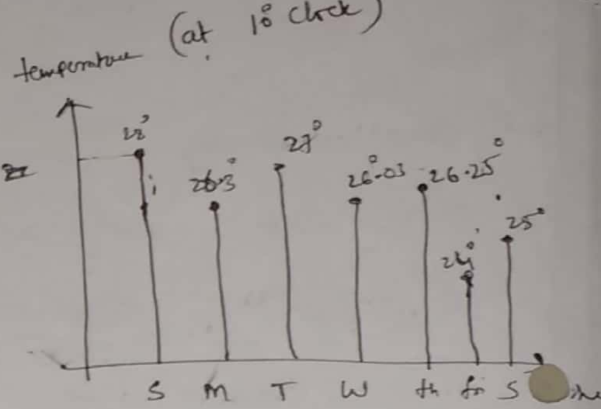
$$x(kT), \quad k = 0, \pm 1, \pm 2, \pm 3 \ldots$$

T → time interval

<u>Example</u>



Continuous. signal



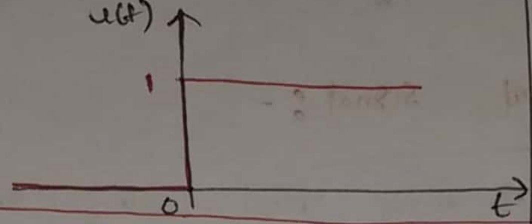
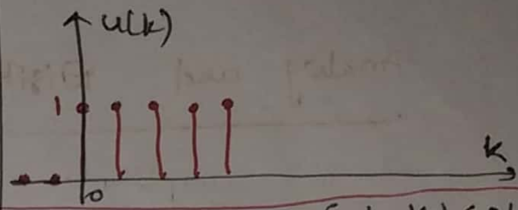
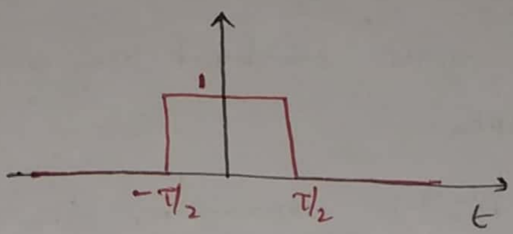
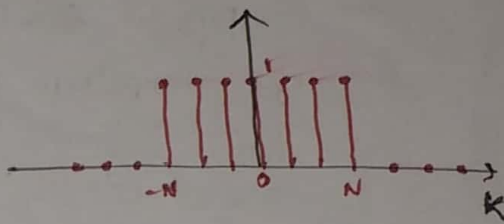
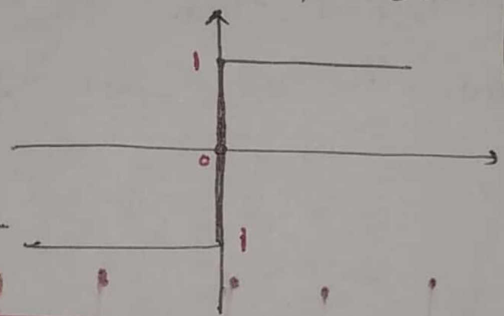
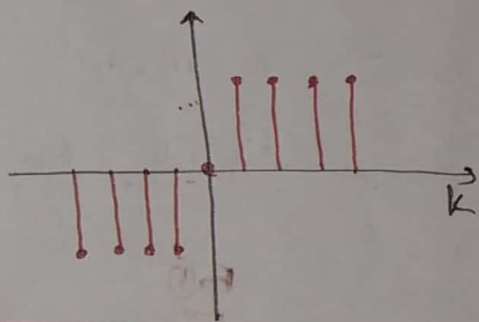
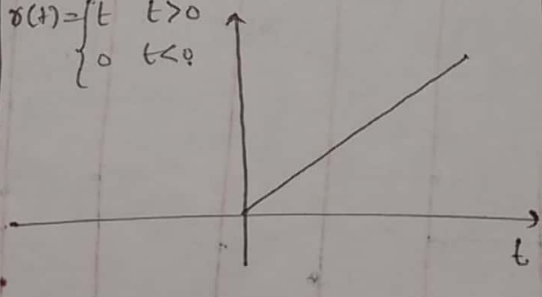
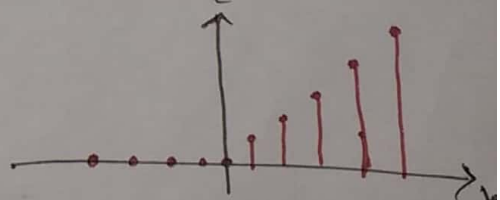
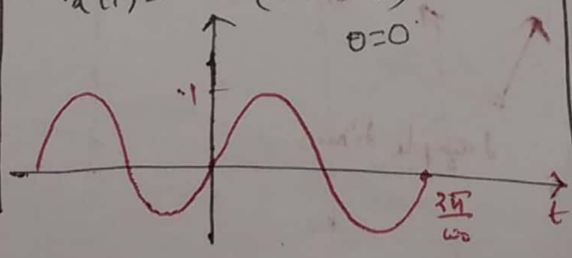
Discrete – time signal.

→ the Continuous – time signal can be converted into Discrete time signal using the sampler.

→ Laplace transform ~~to~~ is used ~~to~~ as a mathematical tool to work with Continuous – time signal. ~~and~~ The systems that process Continuous – time signals are called Continuous – time systems. These systems are modeled mathematically by ~~integro~~ "<u>differential equations.</u>"

→ Z – transform is used as a mathematical tool to work with Discrete – time signal. The Systems that process. Discrete – time signals are called Discrete Systems (Some times Digital Systems). These systems are modeled mathematically by "<u>difference equations</u>"

Some elementary signals. (4) 10/4

	Elementary signals	Continuous-time signal	Discrete-time signal.
1.	unit step	$u(t) = \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$ 	$u(k) = \begin{cases} 1 & k \geq 0 \\ 0 & k<0 \end{cases}$ 
2.	Rectangular pulse	$rect\left(\frac{t}{T}\right) = \begin{cases} 1 & \lvert t \rvert \leq T/2 \\ 0 & \lvert t \rvert > T/2 \end{cases}$ 	$rect\left(\frac{k}{2N+1}\right) = \begin{cases} 1 & \lvert k \rvert \leq N \\ 0 & \lvert k \rvert > N \end{cases}$ 
3.	Signum function	$sign(t) = \begin{cases} 1 & t>0 \\ 0 & t=0 \\ -1 & t<0 \end{cases}$ 	$sgn(k) = \begin{cases} 1 & k>0 \\ 0 & k=0 \\ -1 & k<0 \end{cases}$ 
4.	Ramp function.	$r(t) = \begin{cases} t & t>0 \\ 0 & t<0 \end{cases}$ 	$r(k) = \begin{cases} k & k \geq 0 \\ 0 & k<0 \end{cases}$ 
5.	Sinusoidal function	$x(t) = A \sin(\omega_0 t + \theta)$ $\theta = 0$ 	$x(k) = A \sin(\omega_0 k + \theta)$ 

→ The Discrete signals are represented as

$$\ldots x(-2), x(-1), x(0), x(1), x(2) \ldots x(k) \ldots \quad \text{(or)}$$

$$\{ \ldots x_3, x_2, x_1, x_0 \ldots x_k \ldots \} \quad \text{(or)}$$

↑ (origin)

2. Analog and Digital Signal :-

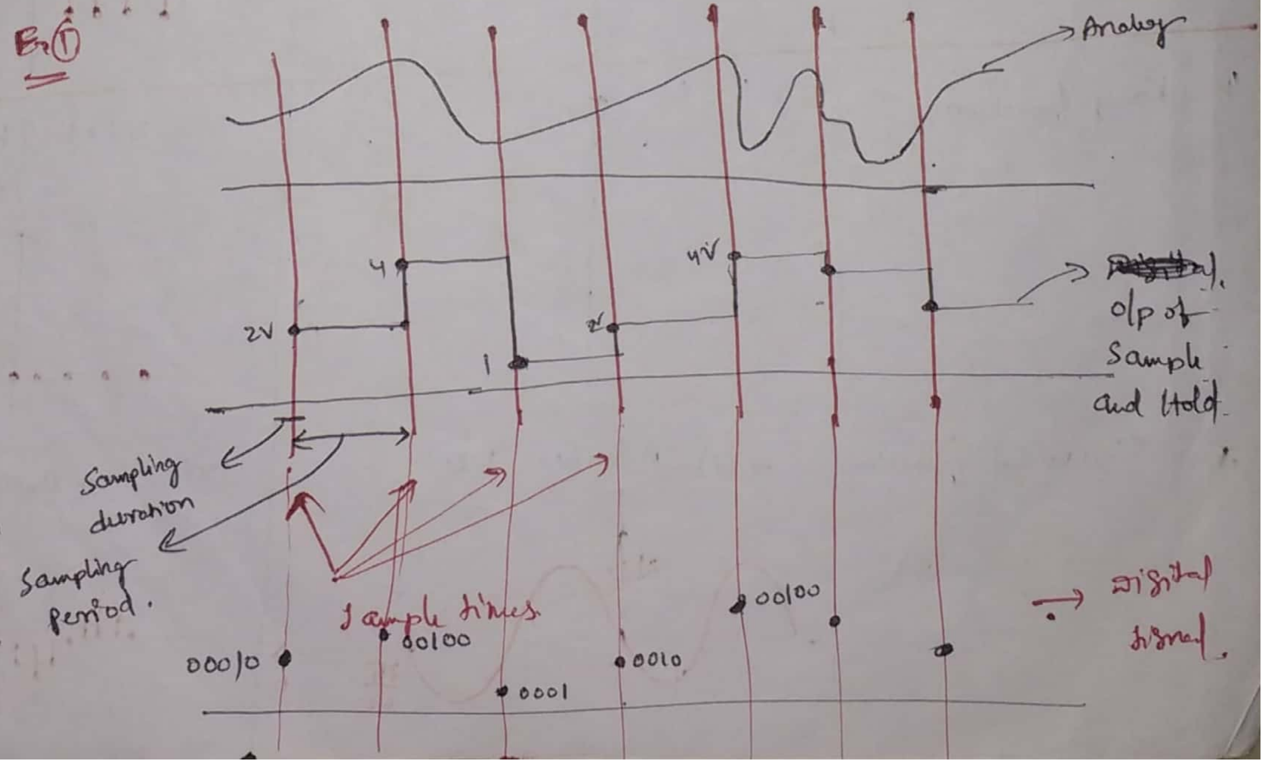
→ The amplitudes of many real-world signals such as voltage, current, temperature, pressure change continuously and these signals are called "Analog signals".

→ Digital signals on the other hand can only have a finite number of Amplitude Values.

Eg: A Digital thermometer with a resolution of $1^{\circ}C$ and a range of $[10^{\circ}C - 30^{\circ}C]$ is used to measure the room temperature at discrete time instants $t = kT$, then the records constitute Digital signal.

→ The Digital Signal is also called "Amplitude quantised discrete signal".

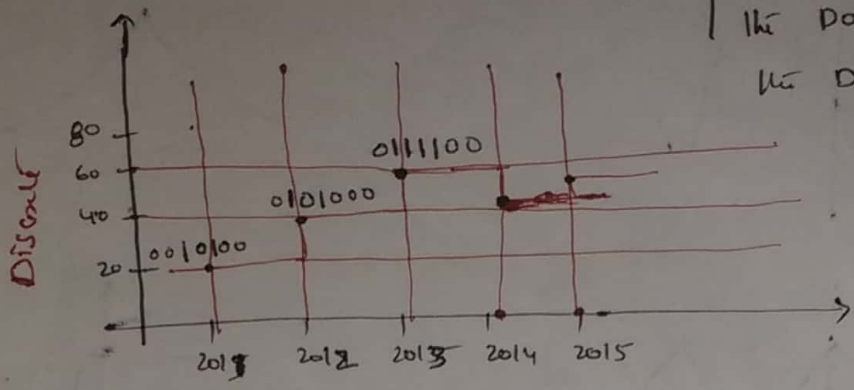
Ex ①



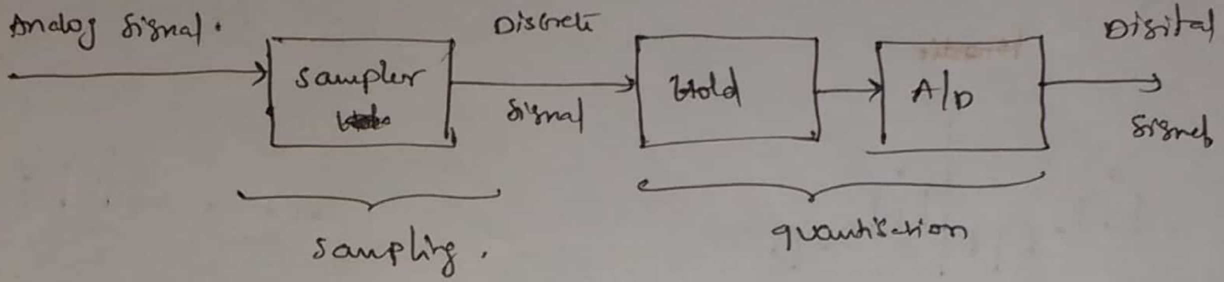
Ex-2 The No. of Employees in an organisation over the past 5 years.

5

[The Dots shown is the Digital signal.]



Summary:-



⇒ Periodic and Aperiodic (or non periodic signals):-

→ A Continuous time signal is said to be periodic if it satisfies the following property

$$x(t) = x(t+T_0) \quad \forall t \quad \& \; T_0 \rightarrow \text{Positive Constant.}$$

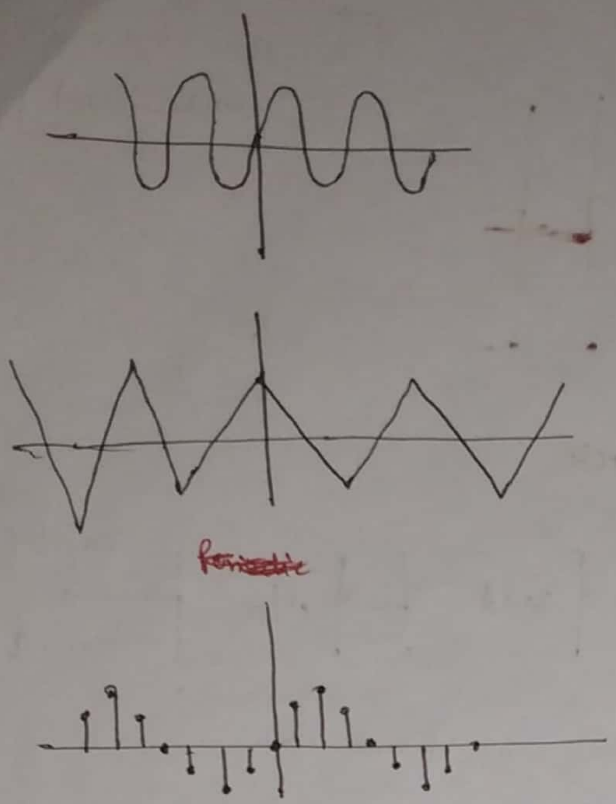
→ The Smallest positive value of T_0 that satisfies the periodicity condition is called "fundamental period of $x(t)$".

→ lly a Discrete time signal is said to be periodic if it satisfies. $x(k) = x(k+k_0)$. $\forall k$, $k_0 \rightarrow$ +ve Constant.

→ The Smallest positive constant k_0 is referred to as the fundamental period.

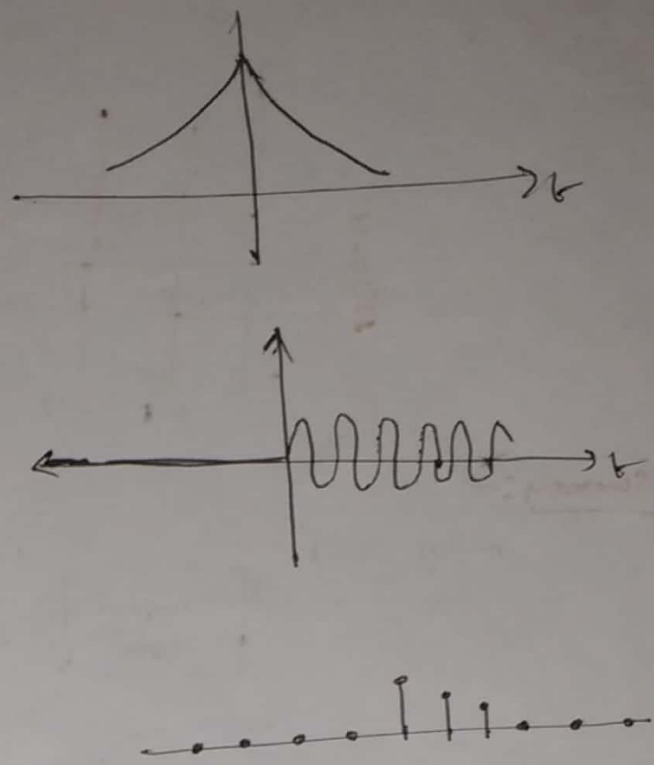
→ A signal which doesn't satisfy the above equations are "non-periodic signals"

Examples



~~Periodic~~

Periodic



Non-periodic

→ The Reciprocal of fundamental period of a signal is called the fundamental frequency.

$$f_0 = \frac{1}{T_0} \quad \text{for CT signal} \qquad f_0 = \frac{1}{K_0} \quad \text{for DT signal}$$

→ The frequency of a signal provides useful information regarding how fast the signal changes its amplitude.

⇒ Linear Combination of two ~~Signals~~ Periodic Signals :—

(6)

→ Let $g(t) = a\,x_1(t) + b\,x_2(t)$

where $g(t)$ is a signal which is linear combination of $x_1(t)$, $x_2(t)$.

→ Then $g(t)$ is periodic iff $\frac{T_1}{T_2} = \frac{m}{n}$ = rational number.

$T_1 \rightarrow$ fundamental Period of signal $x_1(t)$

$T_2 \rightarrow$ fundamental Period of signal $x_2(t)$.

→ The fundamental period of $g(t)$ is given by $nT_1 = mT_2$

Provided that the values of m and n are chosen such that the GCD (greatest Common divisior) between m and n is 1.

Ex $g(t) = 3\sin 4\pi t + 7\cos 3\pi t$ find its fundamental period

Sol. Fundamental period of $3\sin 4\pi t = T_1 = \frac{2\pi}{4\pi} = 1/2$.

" " " $7\cos 3\pi t = T_2 = \frac{2\pi}{3\pi} = 2/3$

$$\frac{T_1}{T_2} = \frac{1/2}{2/3} = 3/4. \leftarrow \text{rational number.} \quad m=3,\ n=4$$

GCD of 3, 4 is 1 so

fundamental period of $g(t) = nT_1 = 4 \times \frac{1}{2} = 2\,sec.$

(or) $mT_2 = 3 \times \frac{2}{3} = 2\,sec$

Ex 2. $g(t) = 3\sin(4\pi t) + 7\cos(10t)$

Sol $T_1 = \frac{2\pi}{4\pi} = 1/2$

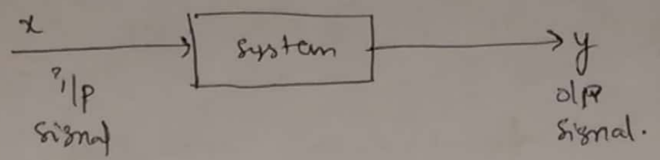
$T_2 = \frac{2\pi}{10} = \pi/5$

$T_1/T_2 = \frac{5}{2\pi}$. Irrational number. ∴ $g(t)$ is non-periodic

Note if the GCD of m and n ≠ 1 then the fundamental period of $g(t)$ = Lcm of m and n.

⇒ Systems and classification of systems :—

→ A system is a mathematical model of a physical process that relates the input (or) excitation signal to the output (or) response signal.

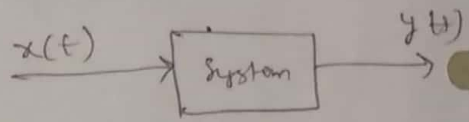


→ **Classification of systems :—**

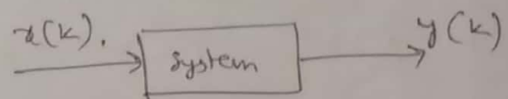
1. linear and non-linear systems.
2. time invariant and time varying systems.
3. systems with and without memory
4. causal and non-causal systems.
5. Invertible and non-invertible systems
6. stable and unstable systems.
7. Continuous-Time and Discrete-time systems.

1. **Continuous-Time & Discrete-time Systems :—**

$x, y \rightarrow$ continuous time signal



$x, y \rightarrow$ discrete time signals



2. **Linear and non-linear systems :—**

Consider a set of inputs and outputs of a single system.

$x_1(t) \longrightarrow y_1(t)$

$x_2(t) \longrightarrow y_2(t)$

then the system is said to be "linear" iff it satisfies the "additivity" and "homogeneity" properties additive property : $x_1(t) + x_2(t) \longrightarrow y_1(t) + y_2(t)$

Homogeneity : $\alpha x_1(t) \longrightarrow \alpha y_1(t)$.

→ additive + Homogeneity = Superposition.

i.e "linear systems satisfy the principle of superposition".

→ Principle of superposition

$$x_1(t) \longrightarrow \boxed{\text{system}(T)} \longrightarrow y_1(t)$$

$$x_2(t) \longrightarrow \boxed{\text{system}(T)} \longrightarrow y_2(t)$$

$$a x_1(t),\ b x_2(t) \longrightarrow \boxed{\text{System } (T)} \longrightarrow a y_1(t) + b y_2(t).$$

→ if the input to a linear system is zero, then the output ~~also be~~ must also be zero for all time t.

this property is called the "zero-input zero-o/p" property

→ A system which donot follow "principle of superposition" is called Non-linear system.

→. the Condition zero-input zero-o/p property is necessary Condition but not sufficient to prove linearity. Many non-linear systems satisfy this property as well.

⇒ Problems check for linearity

① Differentiator $y(t) = \dfrac{dx(t)}{dt}$

Sol

$$x_1(t) \longrightarrow y_1(t) = \frac{dx_1(t)}{dt}$$

$$x_2(t) \longrightarrow y_2(t) = \frac{dx_2(t)}{dt}$$

$$a_1 x_1(t) + a_2 x_2(t) \longrightarrow y'(t) = \frac{d}{dt}\left[a_1 x_1(t) + a_2 x_2(t)\right]$$

$$= a_1 y_1(t) + a_2 y_2(t).$$

∴ System is linear.

(2) $y(t) = e^{x(t)}$

Sol

$x_1(t) \longrightarrow y_1(t) = e^{x_1(t)}$

$x_2(t) \longrightarrow y_2(t) = e^{x_2(t)}$

$a_1 x_1(t) + a_2 x_2(t) \longrightarrow y'(t) = e^{[a_1 x_1(t) + a_2 x_2(t)]}$

$= e^{a_1 x_1(t)} \cdot e^{a_2 x_2(t)}$

$= \left(e^{x_1(t)}\right)^{a_1} \cdot \left(e^{x_2(t)}\right)^{a_2}$

$= [y_1(t)]^{a_1} [y_2(t)]^{a_2}$

$y'(t) \neq a_1 y_1(t) + a_2 y_2(t)$.

[$\because a_1 y_1(t) + a_2 y_2(t) = a_1 e^{x_1(t)} + a_2 e^{x_2(t)}$.]

$\therefore$ Non-linear

(3) $y(t) = 3x(t)$.

Sol

$x_1(t) \longrightarrow y_1(t) = 3x_1(t)$

$x_2(t) \longrightarrow y_2(t) = 3x_2(t)$

$a_1 x_1(t) + a_2 x_2(t) \longrightarrow y'(t) = 3[a_1 x_1(t) + a_2 x_2(t)]$

$y'(t) = a_1 y_1(t) + a_2 y_2(t)$

$\therefore$ linear.

(4) $y(t) = 3x(t) + 5$.

Sol

$x_1(t) \longrightarrow y_1(t) = 3x_1(t) + 5$

$x_2(t) \longrightarrow y_2(t) = 3x_2(t) + 5$

$a_1 x_1(t) + a_2 x_2(t) \longrightarrow y'(t) = 3[a_1 x_1(t) + a_2 x_2(t)] + 5$.

$a_1 y_1(t) + a_2 y_2(t) = 3a_1 x_1(t) + 3a_2 x_2(t) + 10$.

$y'(t) = 3a_1 x_1(t) + 3a_2 x_2(t) + 5$

$y'(t) \neq a_1 y_1(t) + a_2 y_2(t)$

$\therefore$ Non Linear

Pro $y(k) = 3(x(k) - x(k-2))$ ⑧

Sol $x_1(k) \longrightarrow y_1(k) = 3(x_1(k) - x_1(k-2))$

$x_2(k) \longrightarrow y_2(k) = 3(x_2(k) - x_2(k-2))$

$a_1x_1(k) + a_2x_2(k) \longrightarrow y'(k) = 3(a_1x_1(k) + a_2x_2(k) - [a_1x_1(k-2) + a_2x_2(k-2)])$

$= 3[a_1(x_1(k) - x_1(k-2)) + a_2(x_2(k) - x_2(k-2))]$

$y'(k) = a_1y_1(k) + a_2y_2(k)$

$\therefore$ linear.

Pro $y(k) = \sin(x(k))$

Sol $x_1(k) \rightarrow y_1(k) = \sin(x_1(k))$

$x_2(k) \rightarrow y_2(k) = \sin(x_2(k))$

$a_1x_1(k) + a_2x_2(k) \longrightarrow y'(k) = \sin(a_1x_1(k) + a_2x_2(k))$.

but $a_1y_1(k) + a_2y_2(k) = a_1\sin(x_1(k)) + a_2\sin(x_2(k))$

$y'(k) \neq a_1y_1(k) + a_2y_2(k)$ So

Non-linear.

Time Varying and time-Invariant systems:-

→ A system is called time-invariant if a time shift (delay or advance) in the input signal causes the same time shift in the output signal. Thus, ~~for a~~ ~~continuous~~-

if $x(t) \longrightarrow y(t)$ ~~then~~
$x(t-\tau) \longrightarrow y(t-\tau)$ } → time invariant continuous time system.

if $x(k) \longrightarrow y(k)$
$x(k-n) \longrightarrow y(k-n)$ } → time-Invariant Discrete time System.

→ A System which doesn't satisfy the above relations the system is time variant system.

→ A system which is both linear and time-invariant ~~term~~ is called "Linear time invariant system"

→ A system which is both linear and time-varying is called "Liner-time-varying system".

Eg 1 check for time-variant | time invariant. $x(t) \rightarrow \boxed{\text{system}} \rightarrow y(t)$.

① $y(t) = \sin(x(t))$

Sol

$x(t) \longrightarrow y(t) = \sin(x(t))$

$x(t-t_0) \longrightarrow y'(t) = \sin(x(t-t_0))$

$y'(t) = y(t-t_0)$. ✓

∴ time invariant.

② $y(t) = t\sin(x(t))$

Sol

$x(t) \longrightarrow y(t) = t\sin(x(t))$

$x(t-t_0) \longrightarrow y'(t) = t\sin(x(t-t_0))$

but $y(t-t_0) = (t-t_0)\sin(x(t-t_0))$

$y'(t) \neq y(t-t_0)$

∴ time variant system.

③ $y(k) = kx(k)$

Sol

$x(k) \longrightarrow y(k) = kx(k)$

$x(k-k_0) \longrightarrow y'(k) = kx(k-k_0)$

but $y(k-k_0) = (k-k_0)x(k-k_0)$

$y'(k) \neq y(k-k_0)$ so time variant system

(9)

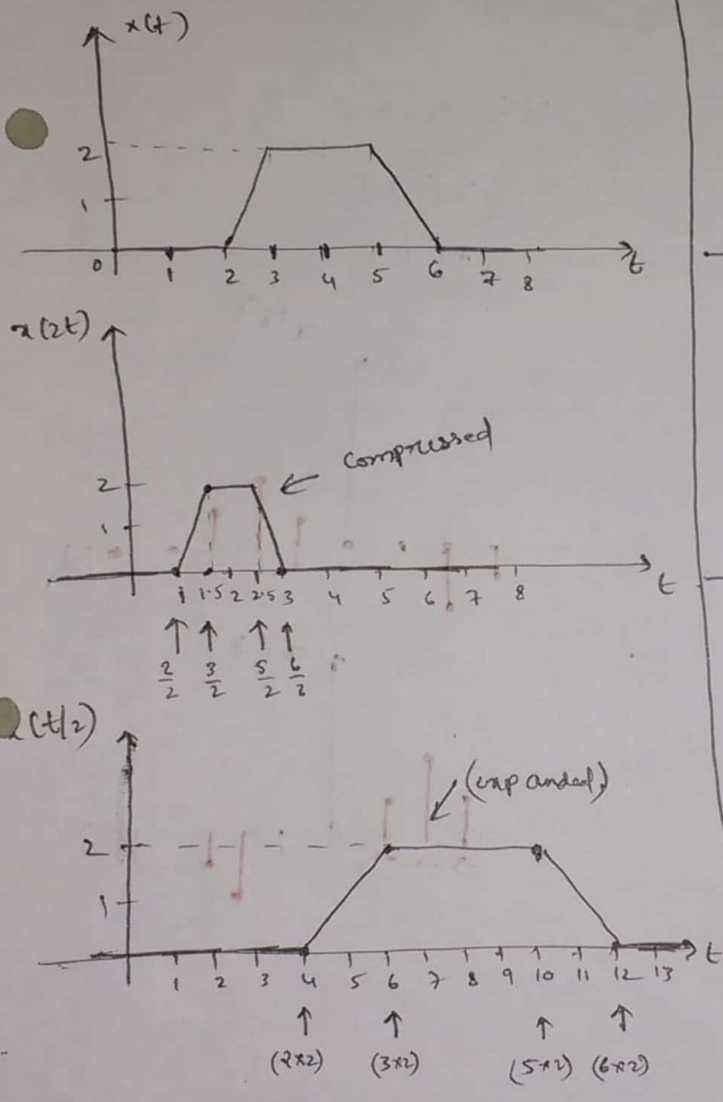
2. Time scaling :-

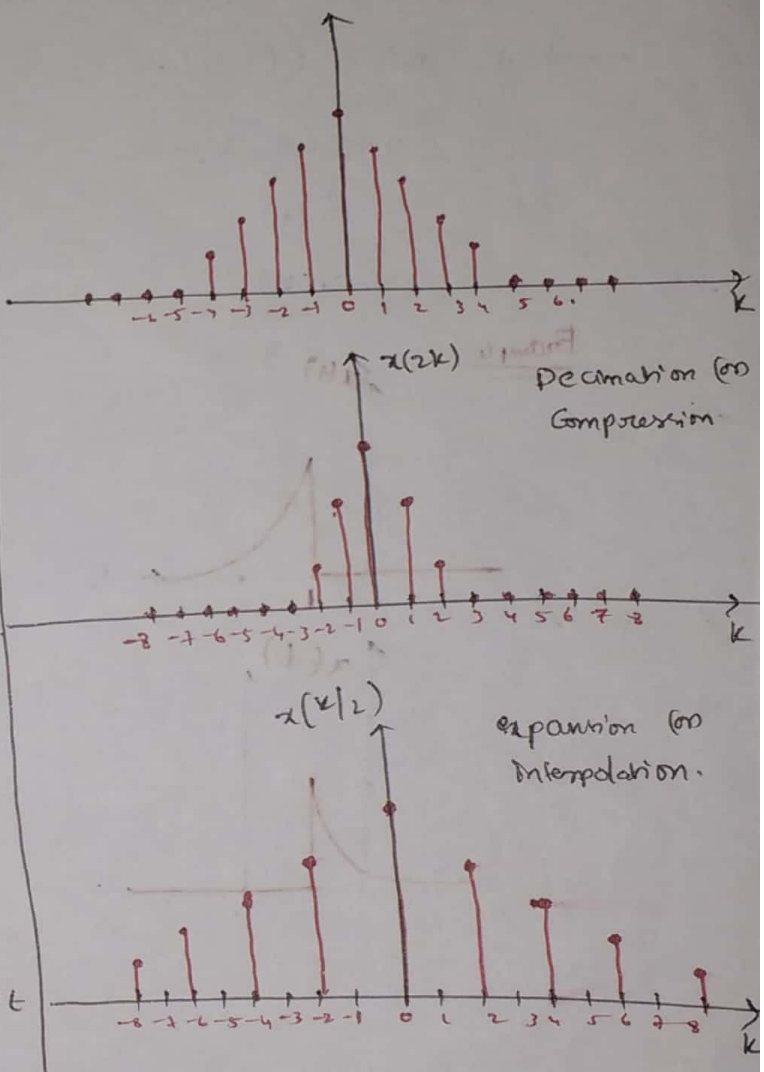
→ Time - scaling compresses or expands the input signals in the time domain.

$$x(t) \xrightarrow[\text{Time-scaled}]{} x(ct)$$

→ if $c > 1$ the signal is compressed by a factor c.
$0 < c < 1$ the signal is expanded by a factor c.

Example





→ ** In the case of Discrete time signals the time - scaling factor 'c' is limited to integers values.

→ for Discrete time signals compression leads to ~~data~~ loss of data samples therefore we cannot recover precisely from the decimated or compressed sequence.

→ Expansion or Interpolation of Discrete - time signals by a factor of m inserts $(m-1)$ zeros in between adjacent samples of the DT sequence.

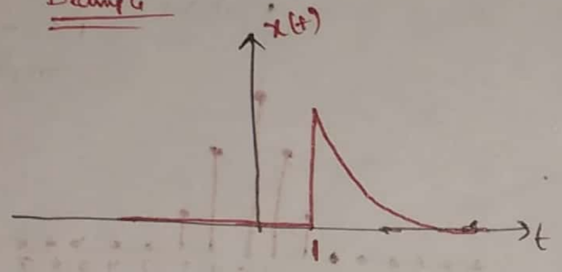
→ Interpolation is a reversible process as the original ~~Process~~ sequence ~~$x(n)$~~ can be recovered.

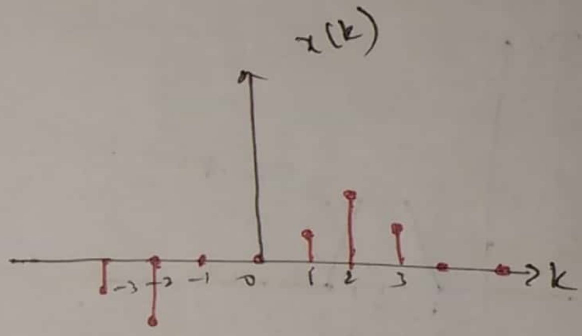
3. Time - Inversion:- (or) Time Reversal (or) Reflection:-

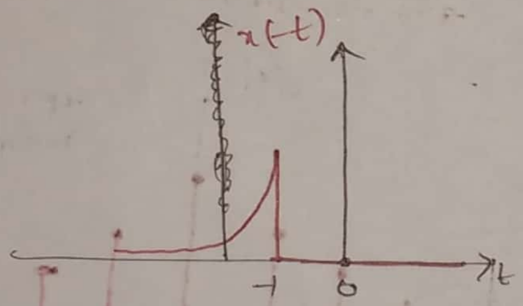
→ Time Reversal operation reflects the input signal about the vertical axis. $(t=0)$.

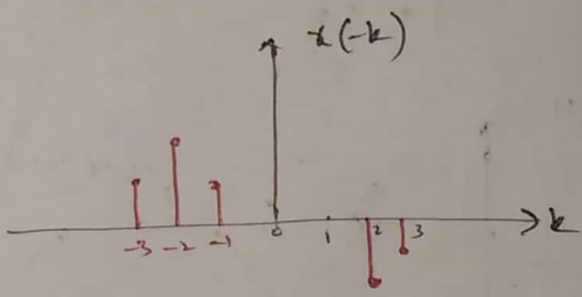
original signal	Time reversal
$x(t)$	$x(-t)$
$x(k)$	$x(-k)$

Example









⇒ ## Combined signal operations :-

if $x(t)$ is original signal then time shifting, time scaling, and reversal, all are represented by $x(\alpha t+\beta)$

→ plot of $x(\alpha t+\beta)$ is done by following steps.

Step-1 : Scale the original signal $x(t)$ by $|\alpha|$. The resulting waveform represent $x(|\alpha| t)$ ($|\alpha|$ → only magnitude)

Step 2 : if α is negative invert the scaled signal $x(|\alpha|t)$ w.r.t $t=0$ axis. This step produces $x(\alpha t)$.

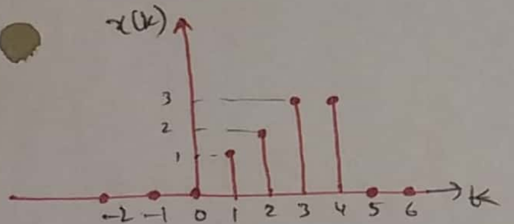
Step 3 : shift the wave form for $x(\alpha t)$ obtained in step 2 by $|\beta/\alpha|$ time-units.

shift Right side if (β/α) is negative.

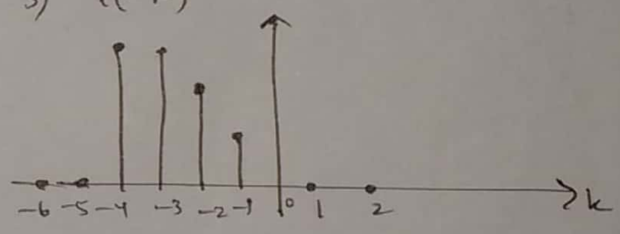
shift left side if (β/α) is positive.

This step completes $x(\alpha t+\beta)$.

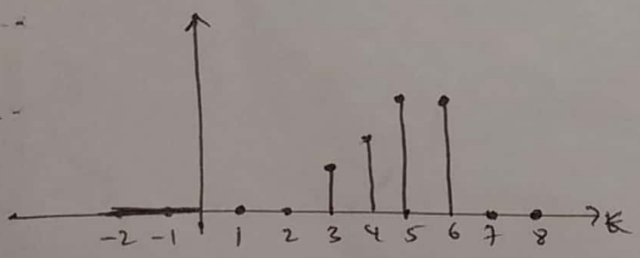
Example :- A Discrete signal $x(k)$ is shown in below figure. Sketch the following 1) $x(k-2)$ 2) $x(2k)$ 3) $x(-k)$ 4) $x(-2k+2)$



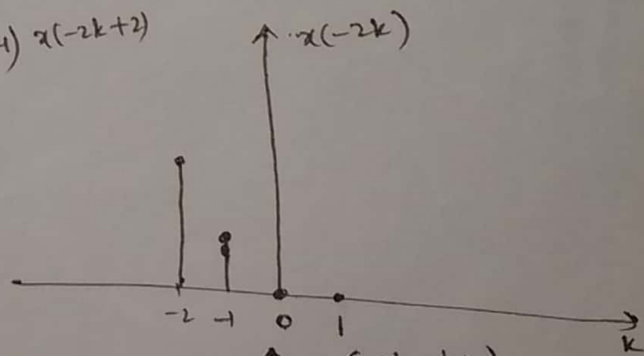
3) $x(-k)$



1) $x(k-2)$ right shift by 2 unit



4) $x(-2k+2)$



2) $x(2k)$